Higher Dimensional Dilaton Black Holes with Cosmological Constant

Chang Jun Gao[1*] Shuang Nan Zhang[1,2,3,4†]

[1]*Department of Physics and Center for Astrophysics, Tsinghua University, Beijing 100084, China(mailaddress)*

[2]*Physics Department, University of Alabama in Huntsville, AL 35899, USA*

[3]*Space Science Laboratory, NASA Marshall Space Flight Center, SD50, Huntsville, AL 35812, USA*

[4]*Laboratory for Particle Astrophysics, Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100039, China*

(February 1, 2008)

Abstract

The metric of a higher-dimensional dilaton black hole in the presence of a cosmological constant is constructed. It is found that the cosmological constant is coupled to the dilaton in a non-trivial way. The dilaton potential with respect to the cosmological constant consists of three Liouville-type potentials.

PACS number(s): 04.20.Ha, 04.50.+h, 04.70.Bw

*E-mail: gaocj@mail.tsinghua.edu.cn

†E-mail: zhangsn@mail.tsinghua.edu.cn

I. INTRODUCTION

There has been much interest in recent years in the dilaton gravity. It is of great importance to investigate how the properties of black holes are modified when the dilaton field is present. Exact solutions of charged dilaton black holes have been constructed by many authors. It is found that dilaton changes the causal structure of the black hole and leads to the curvature singularities at finite radii [1-7]. These black holes are all asymptotically flat. In the presence of one Liouville-type potential which is regarded as the generalization of the cosmological constant, a class of charged black hole solutions have been discovered [8, 9]. Unfortunately, these solutions are asymptotically neither flat nor (anti)-de Sitter.

In fact, Poletti, Wiltshire and Okai [10, 11, 12] have shown that with the exception of a pure cosmological constant, no asymptotically flat, asymptotically de Sitter or asymptotically ant-de Sitter static spherically symmetric solutions to the field equations associated with only one Liouville-type potential exist. In a recent work, we obtained the asymptotically de Sitter and asymptotically anti-de Sitter dilaton solutions in four dimensions [13]. It is found that the dilaton potential which is regarded as an extension of the cosmological constant have three Liouville-type potentials. In this letter, we extend it to arbitrary dimensions.

II. HIGHER DIMENSIONAL DILATON BLACK HOLES WITH COSMOLOGICAL CONSTANT

We consider the n-dimensional theory in which gravity is coupled to dilaton and Maxwell field with an action

$$S=\int d^n x\sqrt{-g}\left[R-\frac{4}{n-2}\partial_\mu\phi\partial^\mu\phi-V\left(\phi\right)-e^{-\frac{4\alpha\phi}{n-2}}F^2\right], \tag{1}$$

where R is the scalar curvature, $F^2=F_{\mu\nu}F^{\mu\nu}$ is the usual Maxwell contribution, and $V\left(\phi\right)$ is a potential of dilaton ϕ which is with respect to the cosmological constant. α is an arbitrary constant governing the strength of the coupling between the dilaton and the Maxwell field.

Varying the action with respect to the metric, Maxwell, and dilaton fields, respectively, yields

$$R_{\mu\nu} = \frac{4}{n-2}\left(\partial_\mu\phi\partial_\nu\phi + \frac{1}{4}g_{\mu\nu}V\right) + 2e^{-\frac{4\alpha\phi}{n-2}}\left(F_{\mu\alpha}F_\nu^\alpha - \frac{1}{2n-4}g_{\mu\nu}F^2\right), \tag{2}$$

$$\partial_\mu\left(\sqrt{-g}e^{-\frac{4\alpha\phi}{n-2}}F^{\mu\nu}\right) = 0, \tag{3}$$

$$\partial_\mu\partial^\mu\phi = \frac{n-2}{8}\frac{\partial V}{\partial\phi} - \frac{\alpha}{2}e^{-\frac{4\alpha\phi}{n-2}}F^2. \tag{4}$$

We choose the most general form of the metric for the static dilaton black hole with a cosmological constant as follows

$$ds^2 = -U(r)\,dt^2 + \frac{1}{U(r)}dr^2 + f(r)^2\,d\Omega_{n-2}^2, \tag{5}$$

where r denotes the radial variable. Then the Maxwell equation Eq.(3) can be integrated to give

$$F_{01} = \frac{Qe^{\frac{4\alpha\phi}{n-2}}}{f^{n-2}}, \tag{6}$$

where Q is the electric charge of the black hole. With the metric Eq.(5) and the Maxwell field Eq.(6), the equations of motion Eqs.(2-4) reduce to three independent equations

$$\frac{1}{f^{n-2}}\frac{d}{dr}\left(f^{n-2}U\frac{d\phi}{dr}\right) = \frac{n-2}{8}\frac{\partial V}{\partial\phi} + \alpha\frac{Q^2e^{\frac{4\alpha\phi}{n-2}}}{f^{2n-4}}, \tag{7}$$

$$\frac{1}{f}\frac{d^2f}{dr^2} = -\frac{4}{(n-2)^2}\left(\frac{d\phi}{dr}\right)^2, \tag{8}$$

$$\frac{1}{f^{n-2}}\frac{d}{dr}\left[U\frac{d}{dr}\left(f^{n-2}\right)\right] = \frac{(n-2)(n-3)}{f^2} - V - 2\frac{Q^2e^{\frac{4\alpha\phi}{n-2}}}{f^{2n-4}}. \tag{9}$$

Since we have no knowledge of how the cosmological constant is coupled to the dilaton, there are four unknown quantities $U(r), f(r), \phi(r)$ and $V(\phi)$ in above three equations of motion. We can not solve them in the usual way. So let's turn our attention to the n-dimensional dilaton black hole solution without the cosmological constant.

The metric for the well-known n-dimensional dilaton black hole without the cosmological constant is given by [14]

$$ds^2 = -\left[1-\left(\frac{r_+}{r}\right)^{n-3}\right]\left[1-\left(\frac{r_-}{r}\right)^{n-3}\right]^{1-\gamma(n-3)} dt^2$$
$$+\left[1-\left(\frac{r_+}{r}\right)^{n-3}\right]^{-1}\left[1-\left(\frac{r_-}{r}\right)^{n-3}\right]^{\gamma-1} dr^2$$
$$+r^2\left[1-\left(\frac{r_-}{r}\right)^{n-3}\right]^{\gamma} d\Omega_{n-2}^2, \tag{10}$$

where r_+, r_- are two event horizons of the black hole and γ is a constant which is related to the coupling constant α and the dimensions n of the spacetime. Here r denotes the radial variable.

Compare Eq.(5) with Eq.(10), we find that $g_{00} \neq -g^{11}$ in Eq.(10) which is different from Eq.(5). To achieve $g_{00} = -g^{11}$, we can rewrite the metric Eq.(10) from the (t, r) coordinate system to the (t, x) coordinate system via the following coordinates transformation

$$x = \int dr\left[1-\left(\frac{r_-}{r}\right)^{n-3}\right]^{-\gamma(n-4)/2}, \quad i.e., \quad r^{'} = \left[1-\left(\frac{r_-}{r}\right)^{n-3}\right]^{\gamma(n-4)/2}, \tag{11}$$

namely, the radial variable r is replaced by x. Here the prime $'$ denotes the derivative with respect to x. Then Eq.(10) is reduced to the following form

$$ds^2 = -\left\{1-\left[\frac{r_+}{r(x)}\right]^{n-3}\right\}\left\{1-\left[\frac{r_-}{r(x)}\right]^{n-3}\right\}^{1-\gamma(n-3)} dt^2$$
$$+\left\{1-\left[\frac{r_+}{r(x)}\right]^{n-3}\right\}^{-1}\left\{1-\left[\frac{r_-}{r(x)}\right]^{n-3}\right\}^{-1+\gamma(n-3)} dx^2$$
$$+r(x)^2\left\{1-\left[\frac{r_-}{r(x)}\right]^{n-3}\right\}^{\gamma} d\Omega_{n-2}^2, \tag{12}$$

where the function $r(x)$ is determined by Eq.(11). It ia apparent $g_{00} = -g^{11}$ in Eq.(12).

Inspecting the four-dimensional dilaton black hole solution with a cosmological constant [13]

$$ds^2 = -\left[\left(1-\frac{r_+}{x}\right)\left(1-\frac{r_-}{x}\right)^{\frac{1-\alpha^2}{1+\alpha^2}} - \frac{1}{3}\lambda x^2\left(1-\frac{r_-}{x}\right)^{\frac{2\alpha^2}{1+\alpha^2}}\right] dt^2$$

$$+\left[\left(1-\frac{r_+}{x}\right)\left(1-\frac{r_-}{x}\right)^{\frac{1-\alpha^2}{1+\alpha^2}}-\frac{1}{3}\lambda x^2\left(1-\frac{r_-}{x}\right)^{\frac{2\alpha^2}{1+\alpha^2}}\right]^{-1}dx^2$$

$$+x^2\left(1-\frac{r_-}{x}\right)^{\frac{2\alpha^2}{1+\alpha^2}}d\Omega_2^2, \tag{13}$$

where λ is the cosmological constant, we suppose the n-dimensional dilaton black hole solution with a cosmological constant has the following form

$$U=\left\{1-\left[\frac{r_+}{r(x)}\right]^{n-3}\right\}\left\{1-\left[\frac{r_-}{r(x)}\right]^{n-3}\right\}^{1-\gamma(n-3)}-\frac{1}{3}\lambda r(x)^2\left\{1-\left[\frac{r_-}{r(x)}\right]^{n-3}\right\}^{\gamma},$$

$$f=r(x)\left\{1-\left[\frac{r_-}{r(x)}\right]^{n-3}\right\}^{\gamma/2}. \tag{14}$$

In the new coordinate system (t,x), the equations of motion Eqs.(7-9) become

$$\frac{1}{f^{n-2}}r'\frac{d}{dr}\left(f^{n-2}Ur'\frac{d\phi}{dr}\right)=\frac{n-2}{8}\frac{\partial V}{\partial\phi}+\alpha\frac{Q^2e^{\frac{4\alpha\phi}{n-2}}}{f^{2n-4}}, \tag{15}$$

$$\frac{1}{f}\frac{d}{dr}\left(r'\frac{df}{dr}\right)=-\frac{4}{(n-2)^2}\left(\frac{d\phi}{dr}\right)^2r', \tag{16}$$

$$\frac{1}{f^{n-2}}r'\frac{d}{dr}\left[Ur'\frac{d}{dr}\left(f^{n-2}\right)\right]=\frac{(n-2)(n-3)}{f^2}-V-2\frac{Q^2e^{\frac{4\alpha\phi}{n-2}}}{f^{2n-4}}, \tag{17}$$

where r denotes $r(x)$.

Substituted the expressions of r' and f into Eq.(16), the dilaton field ϕ is obtained

$$e^{2\phi}=e^{2\phi_0}\left[1-\left(\frac{r_-}{r}\right)^{n-3}\right]^{(n-2)\sqrt{\gamma}\sqrt{2+3\gamma-n\gamma}/2}, \tag{18}$$

where ϕ_0 is the integration constant which has the meaning of the asymptotic value of dilaton.

We in the first place obtain the potential of dilaton $V(\phi)$ from Eq.(17) and then insert both $V(r)$ and $\phi(r)$ into Eq.(15), and then after a lengthy calculation, we find that Eq.(15) is satisfied when

$$\gamma=\frac{2\alpha^2}{(n-3)(n-3+\alpha^2)},$$

$$Q^2=\frac{(n-2)(n-3)^2}{2(n-3+\alpha^2)}e^{-\frac{4\alpha\phi_0}{n-2}}r_+^{n-3}r_-^{n-3}, \tag{19}$$

and then the dilaton potential can be written as

$$
\begin{aligned}
V(\phi) = & \frac{\lambda}{3(n-3+\alpha^2)^2}\Big[-\alpha^2(n-2)\left(n^2-n\alpha^2-6n+\alpha^2+9\right)e^{-\frac{4(n-3)(\phi-\phi_0)}{(n-2)\alpha}} \\
& +(n-2)(n-3)^2\left(n-1-\alpha^2\right)e^{\frac{4\alpha(\phi-\phi_0)}{n-2}} \\
& +4\alpha^2(n-3)(n-2)^2 e^{\frac{-2(\phi-\phi_0)\left(n-3-\alpha^2\right)}{(n-2)\alpha}}\Big].
\end{aligned} \tag{20}
$$

It is clear the cosmological constant is coupled to the dilaton in a very non-trivial way. When $n=4$, Eqs.(19-20) recover our previous results [13].

Come back to the (t,r) coordinate system, we present the metric of a dilaton black hole with the cosmological constant

$$
\begin{aligned}
ds^2 = & -\left\{\left[1-\left(\frac{r_+}{r}\right)^{n-3}\right]\left[1-\left(\frac{r_-}{r}\right)^{n-3}\right]^{1-\gamma(n-3)} - \frac{1}{3}\lambda r^2\left[1-\left(\frac{r_-}{r}\right)^{n-3}\right]^{\gamma}\right\}dt^2 \\
& +\left\{\left[1-\left(\frac{r_+}{r}\right)^{n-3}\right]\left[1-\left(\frac{r_-}{r}\right)^{n-3}\right]^{1-\gamma(n-3)} - \frac{1}{3}\lambda r^2\left[1-\left(\frac{r_-}{r}\right)^{n-3}\right]^{\gamma}\right\}^{-1} \\
& \cdot\left[1-\left(\frac{r_-}{r}\right)^{n-3}\right]^{-\gamma(n-4)}dr^2 + r^2\left[1-\left(\frac{r_-}{r}\right)^{n-3}\right]^{\gamma}d\Omega_{n-2}^2.
\end{aligned} \tag{21}
$$

It is apparent that this spacetime is asymptotically (anti)-de-Sitter. When $\lambda=0$, it restores to the result of Horowitz and Strominger [14]. When $n=4$, it restores to our previous result [13]. When the coupling constant $\alpha=0$, i.e., $\gamma=0$, it restores to the well-known n-dimensional Reissner-Nordstrom-de Sitter metric.

In conclusion, we have constructed the higher-dimensional dilaton black hole solution in the presence of the cosmological constant. So far no fully satisfactory de Sitter version of black hole solutions in string theory has been found [15]. Our solution is asymptotically both flat and (anti)-de Sitter. Thus it may be a fully satisfactory solution of string theory. We found that the dilaton potential with respect to the cosmological constant is not of the form of a pure cosmological constant but the form of three-Liouville-type potential. This is different from the Einstein-Maxwell theory.

ACKNOWLEDGMENTS

This study is supported in part by the Special Funds for Major State Basic Research Projects and by the National Natural Science Foundation of China. SNZ also acknowledges supports by NASA's Marshall Space Flight Center and through NASA's Long Term Space Astrophysics Program.

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